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Technical Note

Micromixing enhanced by pulsating flows

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ABSTRACT

We consider the micromixing enhancement by pulsating flows. Dimensionless governing equations and boundary conditions were developed for T-type micromixers with two inlet pulsating flows. The problem involves a set of parameters. Three key dimensionless parameters are identified: the Reynolds number, the Strouhal number, and the disturbance amplitude. Suitable Strouhal number or disturbance amplitude causes symmetrical meniscus-shape mixing interfaces, separating the whole mixing channel into a set of segments. Thus uniform exit species concentration can be reached. Too large or too small Strouhal number or disturbance amplitude yields the meniscus-shape mixing interfaces deviating from the centerline of the mixing channel, deteriorating the mixing performance. The optimized disturbance amplitude is increased with increases in Strouhal numbers. Low Reynolds number needs larger disturbance amplitude.

1. Introduction

Micromixing becomes difficult due to the lack of turbulence for low Reynolds number flows in microscale [1]. It can only be fulfilled by molecular diffusion, behaving very slowly. Inlet pulsating flows can enhance the micromixing process. This concept of mixing was first proposed by Volpert et al. [2]. Bottausci et al. [3] designed a micromixer with three cross-stream secondary channels, providing time-dependent actuation of the flow stream perpendicular to the main stream. It is shown that mixing was substantially improved with oscillatory flows in multiple side channels. Cross-channel micromixers were reported in [4–6], in which the main stream was perturbed by an external oscillatory-flow excitation in side channel. Adjusting amplitude and frequency of the oscillatory flow, the wavy and chaotic regimes were obtained. A spatiotemporal resonance phenomenon was observed in their experiments. It is found that the resonant states could be used to separate particles of different sizes. Niu and Lee [7] studied chaotic behaviors in a micromixer with multiple side channels stirred by oscillatory flows. They found that Lyapunov exponent is closely related to the amplitude and frequency of stirring and can be used to optimize design and operation of micromixers.

Nguyen and Huang [8,9] reported a micromixer with pulsating flows at the entrances using two piezoelectric valves. Truesdell et al. [10] proposed the pulsation action to mix two streams entering a tube from two separated branches of a bifurcation at low Reynolds numbers. The two pinch valves used in their experimental setup deform the flexible tubing immediately upstream of the connection. The mixing quality was evaluated by measuring the fluorescence resulting from the chemical reaction of species transported in the two streams. The results showed that the mixing process was almost complete. Glasgow and Aubry [11] studied the enhanced micromixing by the computational fluid dynamics (CFD). All channels were 200 μm wide by 120 μm deep. They found that the best results occurred when both inlets were pulsed out of phase. The interface was shown to stretch, retain one fold, and sweep through the confluence zone, leading to good mixing within 2 mm downstream of the confluence, i.e., about 1 s of contact. A simple time-dependent analytical solution for the pulsating flow enhanced micromixing was proposed by Nguyen and Huang [12].

In this paper, we study the micromixing enhancement by pulsating flows. The dimensionless governing equations and boundary conditions are developed. Three key parameters were identified. The optimal parameters for the micromixing enhancement were discussed.

2. Numerical simulation

T-type micromixers are used, whose dimensions and coordinates are shown in Fig. 1. The original point (0, 0, 0) is at the confluence center between the two inlet channels. The x-coordinate is at the mixing channel centerline. The y-coordinate is the chip width direction perpendicular to the x-coordinate. The z-coordinate refers to the channel depth direction. The mixing channel has the length of L, with the channel width of W_c and depth of

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Nomenclature

a	disturbance amplitude
b	velocity ratio between two average inlet velocities
c	species concentration (molar/l)
D	molecular diffusion ($m^2\,s^{-1}$)
D_h	hydraulic diameter (m)
f	pulsating frequency (Hz)
H	microchannel depth (m)
L	length of the mixing channel (m)
$\overline{MD}$	mixing degree
N	number of grid cells
p	pressure (Pa)
P	dimensionless pressure
Pe	Peclet number
Re	Reynolds number
S	half distance between the two fluid inlets (m)
Sc	Schmidt number
St	Strouhal number
t	times (s)
u, v, w	velocity in x-, y- and z-coordinates (m/s)
U, V, W	dimensionless velocities in X-, Y- and Z-coordinates
$\bar{v}$	average inlet velocity (m/s)
W_c	microchannel width (m)
x, y, z	three coordinates shown in Fig. 1 (m)
X, Y, Z	three dimensionless coordinates

Greek symbols

ρ	density (kg/m^3)
ν	kinematic viscosity (m^2/s)
τ	dimensionless time
ψ	dimensionless concentration
φ	phase angle difference between two inlet fluids

Subscripts

1	value of fluid 1
2	value of fluid 2

H. The two inlet channels have the same cross section as that of the mixing channel, having the total length of 2S. An example design is L = 6.0 mm, W_c = 200 μm, H = 40 μm, S = 2.0 mm. Two aqueous species termed as fluid 1 and fluid 2 enter the T-mixer at their entrances. The inlet concentrations are 1.0 M for the fluid 1 and 0.0 M for the fluid 2. In the ideal situation, the concentration should be 0.5 M at the mixing channel exit with the same flow rate of the two inlet fluids. At room temperature, the fluid kinematic viscosity (ν) is about $10^{-6}\,m^2\,s^{-1}$. The molecular diffusivity coefficient (D) is in the order of $10^{-10}\,m^2\,s^{-1}$ for most biomolecules, such as RNA, DNA, and proteins. The conservation mass, momentum, and species concentration equations follow those described in Ma et al. [13].

The inflow conditions for pulsating flows are written as

$$v_i = \bar{v}_i[1 + a_i \sin(2\pi f_i t + \varphi_i)] \quad (1)$$

where φ_i is the phase angle difference between the two pulsating flows. For fluid 1, φ_i is zero.

Introducing the following dimensionless parameters:

$$X = \frac{x}{D_h}, \quad Y = \frac{y}{D_h}, \quad Z = \frac{z}{D_h}, \quad \tau = \frac{t(\bar{v}_1 + \bar{v}_2)}{D_h}, \quad P = \frac{p}{\rho(\bar{v}_1 + \bar{v}_2)^2} \quad (2)$$

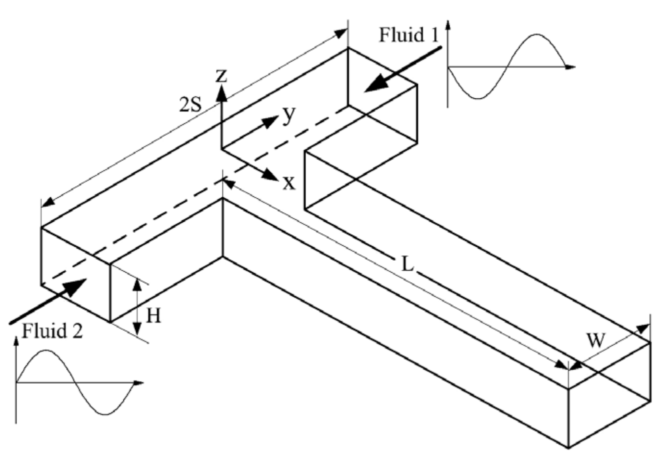

Fig. 1. The geometry of the T-type mixer and the coordinate system.

$$U = \frac{u}{(\bar{v}_1 + \bar{v}_2)}, \quad V = \frac{v}{(\bar{v}_1 + \bar{v}_2)}, \quad W = \frac{w}{(\bar{v}_1 + \bar{v}_2)},$$
$$Re = \frac{(\bar{v}_1 + \bar{v}_2)D_h}{\nu}, \quad Pe = \frac{(\bar{v}_1 + \bar{v}_2)D_h}{D}, \quad \Psi = \frac{c}{c_1} \quad (3)$$

The three conservation equations are modified to be

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \quad (4)$$
$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} + W\frac{\partial U}{\partial Z} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\nabla^2 U \quad (5)$$
$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} + W\frac{\partial V}{\partial Z} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\nabla^2 V \quad (6)$$
$$\frac{\partial W}{\partial \tau} + U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} + W\frac{\partial W}{\partial Z} = -\frac{\partial P}{\partial Z} + \frac{1}{Re}\nabla^2 W \quad (7)$$
$$\frac{\partial \Psi}{\partial \tau} + U\frac{\partial \Psi}{\partial X} + V\frac{\partial \Psi}{\partial Y} + W\frac{\partial \Psi}{\partial Z} = \frac{1}{Pe}\left(\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} + \frac{\partial^2 \Psi}{\partial Z^2}\right) \quad (8)$$

The Strouhal number is the ratio of the characteristic flow time to the pulsating time period:

$$St_1 = \frac{f_1 D_h}{\bar{v}_1 + \bar{v}_2} = \frac{D_h^2}{\nu Re} f_1, \quad St_2 = \frac{f_2 D_h}{\bar{v}_1 + \bar{v}_2} = \frac{D_h^2}{\nu Re} f_2 \quad (9)$$

The Peclet number (Pe) is not an independent parameter, but depends on the Schmidt number and the Reynolds number, i.e., Pe = Sc· Re, indicating 10^4 times of the Reynolds number for the fluid properties. The ratio of the average velocity of fluid 1 to that of fluid 2 is defined as $b = \bar{v}_1/\bar{v}_2$.

The inlet boundary conditions are

$$U = W = 0, \quad V = -\frac{b}{1+b}[1 + a_1 \sin(2\pi St_1 \tau)],$$
$$\Psi = 1 \qquad \text{at } Y = S/D_h \quad (10)$$

$$U = W = 0, \quad V = \frac{1}{1+b}[1 + a_2 \sin(2\pi St_2 \tau + \varphi)],$$
$$\Psi = 0 \qquad \text{at } Y = -S/D_h \quad (11)$$

On the channel wall surfaces, we use the no-slip and no-penetration boundary conditions, i.e., $U = V = W = 0$, $\frac{\partial \Psi}{\partial \bar{n}} = 0$, where $\bar{n}$ is perpendicular to the side walls of the channels.

Eqs. (4)–(8), (10) and (11) involve a set of dimensionless parameters of W_c/D_h, L/D_h, S/D_h for the T-mixer dimensions, Re for the Reynolds number, Pe for the Peclet number, a_1, a_2, b, φ, St_1, St_2

for the inlet boundary conditions. Once *Re* is given, *Pe* can be determined.

Selection of φ: Theoretically φ can be any values. The backflow depends not only on φ but also on a_1 and a_2. For the present simulations, φ equals to π for a_1 and a_2 larger than unity. This promotes the micromixing process, as noted in [11].

Selection of Re: In the present study, Reynolds numbers are selected as 0.1, 1 and 10.

Selection of St or f: Similar to φ, unsuitable Strouhal numbers may cause the backflow in the mixing channel. For *Re* of 0.1, 1 and 10, the average velocities in the mixing channel are 1.5, 15 and 150 mm/s, respectively. Because the fluid residence time in the mixing channel should be at least two full pulsating time periods, the average velocity and the pulsating frequency shall satisfy the criterion of $Lf/(\bar{v}_1 + \bar{v}_2) > 2$, corresponding to $f > 0.5$ Hz for $Re = 0.1$, $f > 5$ Hz for $Re = 1.0$, and $f > 50$ Hz for $Re = 10$.

Similar to the phase angle difference (φ), the disturbance amplitudes of the two inlets (a_1 and a_2) should be identical. The ratio of the two average velocities (b) is chosen to be unity.

Due to the above analysis, the present investigation studies effects of *Re*, *St* and *a*.

The mixing degree represents how well the two fluids are mixed in the mixing channel. The cycle averaged mixing degree is defined as [13]

$$\overline{MD} = AVE\left(1.0 - \frac{1}{\bar{c}}\sqrt{\frac{\sum_{i=1}^{N}(c_i - \bar{c})^2\left(\frac{V_i}{\overline{V}}\right)}{N}}\right) \tag{12}$$

where $\bar{c} = 0.5$ M is the output concentration for an ideal mixer and $\overline{V}$ is the average velocity. The zero of $\overline{MD}$ indicates no mixing in the channel, while the value of 1 means the complete mixing.

The hexahedron grid cells are used. The sensitivity analysis was performed. Numerical computations of mixing process at high Peclet numbers are often inaccurate due to the numerical diffusion. The number of grid cells is 1.2 million in this study. For a case of $Re = 10$, $f = 200$ Hz and $a_1 = a_2 = 12.5$, the mixing degree at the mixing channel exit is 0.906. Such a mixing degree becomes 0.903 if the grid cells are doubled, indicating the number of grid cells is large enough for the computation.

The FLUENT 6.0 software package is used. In each time step, the momentum equation and species concentration equation are iterated alternatively, until the convergence solution is reached. Fifty time steps are applied in one pulsating cycle. For the momentum equation, the second-order accuracy in space is enough. For the species concentration equation, the third-order QUICK scheme is used. The first-order implicit scheme is used for time-stepping and SIMPLE algorithm for pressure-correction.

Nguyen and Huang [12] gave a simplified one-dimension semi-analytical solution expressed in a series expansion for the species concentration along the flow direction, using the fully developed Poiseuille velocity profile depending on the channel depth. It is found that our numerical simulations agree with the semi-analytical solution given in [12]. The reason for the small deviation between numerical solution and the semi-analytical model is that the semi-analytical model overestimates the effective diffusion efficient in the axial direction.

3. Results and discussion

The disturbance amplitude significantly influences the mixing performance. During the pulsating flow, the whole mixing channel is separated by a set of meniscus-shape mixing interfaces. The segmentation number is the number of sections separated by meniscus-shape interfaces. At $Re = 10$, $f = 10$ and 100 Hz, the disturbance amplitude of 5 yields five segments separated by four meniscus-shape mixing interfaces. The meniscus-shape mixing interfaces deviate from the mixing channel centerline. The tip of the meniscus-shape interface approaches the front and rear side walls of the mixing channel alternatively. At the mixing channel exit, the concentration is higher near the rear side wall, corresponding to the rear inlet channel of fluid 1.

The mixing performance is improved if the disturbance amplitude is increased to 10. The meniscus-shape mixing interfaces are almost symmetry against $Y = 0$, corresponding to a more uniform concentration at the mixing channel exit. If the disturbance amplitude is too large such as $a_1 = a_2 = 20$ also yields the poor mixing performance. Again, the meniscus-shape mixing interfaces deviate from the axial centerline of the mixing channel.

The Strouhal number (or the pulsating frequency) is another important parameter to affect the mixing performance. The low pulsating frequency of 5 Hz involves three segments in the mixing channel, corresponding to the mixing degree of 0.739 at the mixing channel exit. Due to the less number of separated segments in the mixing channel, the interface area is not large enough to achieve good mixing. When the pulsating frequency is increased to 10 Hz, there are five segments in the mixing channel, with the mixing degree of 0.864 at the channel exit, enhancing the mixing of two fluids significantly. The meniscus-shape mixing interfaces are symmetrically populated along the flow direction of the mixing channel, causing a uniform concentration distribution at the mixing channel exit. Similar to the effect of the disturbance amplitudes, much higher pulsating frequency of 20 Hz causes asymmetrical meniscus-shape mixing interfaces, deteriorating the mixing performance.

The mixing performance can be improved by increasing the Reynolds number for the steady flow. However, this is not true for the unsteady pulsating flow. We fix $a_1 = a_2 = 20$, $St_1 = St_2 = 0.0445$, but the Reynolds number are 0.1, 1 and 10, respectively. For the low Reynolds number of 0.1, the two fluids are nearly thoroughly mixed after three segments along the axial flow direction, with the mixing degree of 0.954 at the exit cross section. For the Reynolds number of 1, the fluids are well mixed after five segments passed, with the mixing degree of 0.864 at the outlet cross section. The higher Reynolds number of 10 leads to the meniscus-shape mixing interfaces deviating from the centerline of the mixing channel, yielding the apparent concentration gradient in the *Y*-direction at the exit cross section. The exit mixing degree is 0.681 for such run. The decreased mixing performance with increasing the Reynolds number is due to the decreased residence time of fluid samples in the mixing channel. Even though the decreased mixing performance when Reynolds number is increased, the pulsating flow is very useful for the mixing enhancement. This is because the steady flow without pulsating flow gets very poor mixing performance. For instance, the mixing degree at the channel exit is 0.053 when the Reynolds number is 1 for the steady flow.

Fig. 2 shows the mixing degree varied in different cross sections along the flow direction. The flow length dependent mixing degree is integrated in a full pulsating time period. The mixing degree is found to be increased quickly for $x < 3$ mm, indicating the convection dominated diffusion caused by the pulsating flow in the mixing channel upstream. The mixing degree does not change apparently for x larger than 3 mm, indicating the molecular diffusion in the mixing channel downstream. Again, we observed that the low Reynolds number of 0.1 possesses better mixing performance than those for other two runs.

Now we study the relationship among the Reynolds number, the Strouhal number, and the disturbance amplitude. We define the optimized disturbance amplitude which could result in the symmetry meniscus-shape mixing interfaces in the channel and

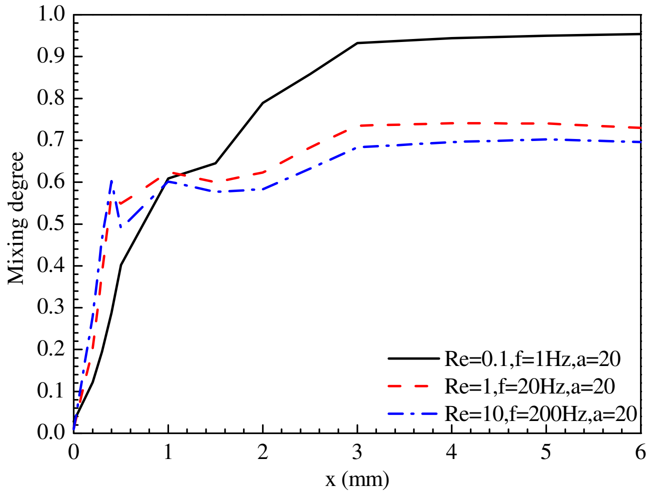

Fig. 2. The cross-sectional averaged mixing degree along the axial flow direction.

the maximum outlet mixing degree. Fig. 3 gives such relationship. The general trend is that the lower Reynolds number needs larger disturbance amplitude. Besides, the optimized disturbance amplitudes are increased with increasing the Strouhal numbers, especially for the Reynolds number of 0.1 and 1.

The exit mixing degrees could reach high values with the Strouhal number in the range of 0.0455–0.0899 for Reynolds numbers from 0.1 to 10. The number of separated segments in the mixing channel depends on $L \cdot St/D_h$. There are 4–8 segments in the mixing channel with the above Strouhal number range. A set of segments provide large interface area for the mixing enhancement. Besides, a specific combination of the Reynolds number and Strouhal number, there is a optimized disturbance amplitude maximally enhancing the mixing process between two fluids. Too large or too small disturbance amplitudes cause the meniscus-shape mixing interfaces deviating from the centerline of the mixing channel, deteriorating the mixing process in the channel.

Nguyen and Wu [1] gave a mixing map with Strohal number and Reynolds number as coordinates (see Fig. 12 in Ref. [1]). They indicate that higher Reynolds number needs a larger Strohal number. Truesdell et al. [10] introduced pulsating flows through a branch channel, which is different from the present simulation in

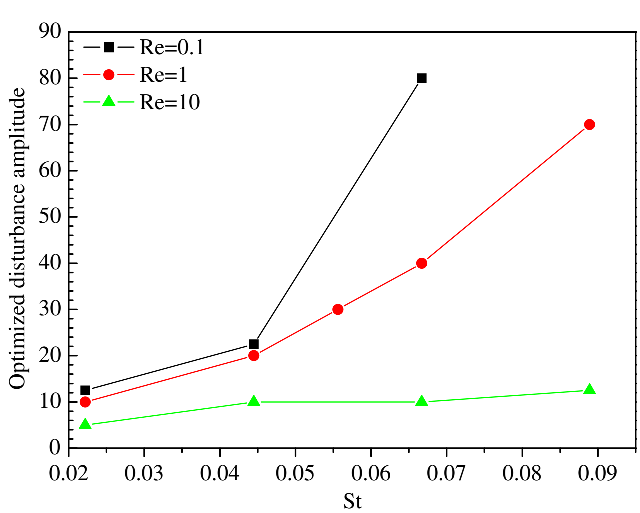

Fig. 3. The optimized disturbance amplitude changed with the Strouhal number and Reynolds number.

which pulsating flows are introduced directly from the fluid source. Ma et al. [13] studied the pulsating flow enhanced mixing in microchannels by employing linear superimposition of velocity fields with different combinations of mean input flow and pressure difference. They found that the mixing degree can heave optimal values at Strohal number of 0.42 for Reynolds number less than 0.24. Too small or large Strohal numbers deteriorate the mixing performance. In this paper we gave a simple mixing map in Fig. 3. The findings are consistent with those reported in Refs. [1,10,13], qualitatively. Because experimental and/or numerical technical details of this paper are different from those in refs. [1,10,13] to some extent, direct comparisons with those reported in Refs. [1,10,13] are not convenient.

4. Conclusions

We consider the micromixing enhancement by pulsating flows. Dimensionless governing equations and boundary conditions were developed for T-type micromixers with two inlet pulsating flows. The problem involves a set of parameters. Three key dimensionless parameters are identified: the Reynolds number, the Strouhal number, and the disturbance amplitude. Suitable Strouhal number or disturbance amplitude causes symmetrical meniscus-shape mixing interfaces, separating the whole mixing channel into a set of segments. Thus uniform exit species concentration can be reached. Too large or too small Strouhal number or disturbance amplitude yields the meniscus-shape mixing interfaces deviating from the centerline of the mixing channel, deteriorating the mixing performance. The optimized disturbance amplitude is increased with increases in Strouhal numbers. Low Reynolds number needs larger disturbance amplitude.

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